# **Mathematical Modeling and Analysis**



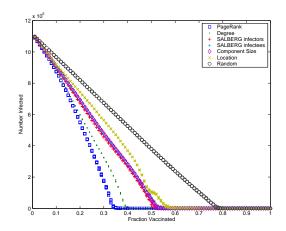
## Preventing or Reducing the Spread of Epidemics

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Recent events have highlighted the need to contain the spread of infectious diseases. Conventionally we attempt to control many of these diseases through vaccination. However, a number of factors can affect the effectiveness of a vaccination campaign. Three particular concerns emerge from recent headlines. SARS demonstrated that new diseases can emerge for which a vaccine is not initially available. Bioterrorism raises the threat of a smallpox outbreak in a population largely unvaccinated where logistical problems may complicate a sudden large-scale vaccination campaign. The unexpected shortage of flu vaccine this year as well as the reduced effectiveness of the vaccine last year show that even wellknown diseases can strain our vaccination campaigns.

One basic question motivates this work: "given a limited vaccine supply, how should it be distributed to minimise the number of people infected?" We should note that this does not form the only criterion for planning a vaccination campaign. For example, some diseases affect the elderly or the young more significantly than the general population, and so consideration should go to preferentially vaccinating those most at risk rather than those most likely to spread the disease. In this work we focus only on reducing the number of people infected, ignoring these complications.

In order to compare different vaccination strategies, we need a model of the social interactions within a population on which to run simulations of the disease. For this we use a network model provided by Episims. Once we have a suitable model, we need to investigate how the disease spreads in the presence of different vaccination strategies. In the final analysis, we must consider a cost-benefit analysis of the additional cost of implementing a sophisticated strategy in light



A comparison of different vaccination strategies

of the savings in number infected.

#### The Social Model

In order to get an accurate idea of how the disease is spreading, we need an accurate model of the social interactions. A number of models have been developed, with a wide range of simplifying assumptions. One of the most sophisticated is Episims [1], a computer simulation of the city of Portland, Oregon. This simulation consists of approximately 1.6 million people who travel through the city on foot, in cars, in public transportation while going to work, school or carrying out tasks such as shopping. The population is based on census data, and the simulated city contains the transportation network of the physical city.

The simulation provides a wide range of data. Most significant for the purposes of studying disease spread, it tells us what people are at the same place at the same time. Given this data and information about the infectiousness of a disease, we can simulate epidemics.

### The Vaccination Strategies

We consider a number of vaccination strategies

• Random Vaccination: The most obvious way to implement a vaccination strategy is to simply vaccinate a number of people without any concern for their behavior. This is the least sophisticated strategy and provides a baseline for comparing the other strategies.

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- *Vaccination by degree*: One of the most obvious ways to prioritise the vaccinations is by degree (the number of contacts a person has).
- Vaccination by locations visited: Another obvious way to prioritise vaccinations is by the number of locations a person visits.
- Vaccination by component size: If an epidemic starts with a single individual, it will infect some number of people. The expected final number may depend on which person was the initial infection. It seems reasonable that if we vaccinate those people who result in the most infections, we will reduce the final size of an epidemic.
- Vaccination by SALBERG rankings: A more sophisticated algorithm comes from a hybrid of Kleinberg's HITS algorithm [2] and Lempel and Moran's SALSA algorithm [3]. This algorithm ranks people by some measure of their ability to infect others as well as by some measure of their likelihood to be infected during the epidemic.
- Vaccination by PageRank: Another related algorithm is Brin and Page's PageRank algorithm [4] which forms the basis of Google's ranking algorithm.

To compare these strategies, we vaccinated a percentage of the population according to the rankings provided by each strategy. We ran a number of simulations on the remaining population, and looked at the average outbreak size. The resulting sizes are shown in the figure.

### Conclusions and Future Work

The figure shows that any amount of targetting provides a substantial improvement over random vaccination. However, the best improvement comes from the PageRank algorithm, which slightly outperforms vaccination by degree.

Vaccination by degree is a fairly simple algorithm to implement. It depends simply on knowing how many people an individual meets. In contrast, the PageRank algorithm depends on knowing not just who an individual meets, but who

his acquaintances meet and in turn their acquaintances, etc. This is likely to be impractical in a real-world environment where such information is likely to be unavailable.

We have shown that in the population produced by Episims, there are vaccination strategies which improve on simply vaccinating those people who have the most contacts. Although the PageRank algorithm may not be practical in the real world, it does raise the possibility that other algorithms are out there which improve on what is widely treated as the optimal strategy. This suggests a need for more research into alternative ways to improve vaccination strategies. It also suggests the need for research into other networks to determine if the PageRank advantage persists, or if it is perhaps an artifact of some irregularity of the Episims network.

#### References

- [1] S. EUBANK, H. GUCLU, V. S. A. KUMAR, M. V. MARATHE, A. SRINIVASAN, Z. TOROCZKAI, AND N. WANG. Modelling disease outbreaks in realistic urban social networks. *Nature*, 429:180–184, 2004.
- [2] J. KLEINBERG. Authoritative sources in a hyperlinked environment. *Journal of the ACM*, 46:604–632, 1999.
- [3] R. LEMPEL AND S. MORAN. The stochastic approach for link-structure analysis (SALSA) and the TKC effect. In Proceedings of the Ninth International Conference on the World Wide Web, May 2000.
- [4] S. BRIN AND L. PAGE. The anatomy of a large-scale hypertextual (web) search engine. In *Proceedings of* the Seventh International Conference on the World Wide Web, 1998.